OBJECTIVE MATHEMATICS

Volume 1

Descriptive Test Series

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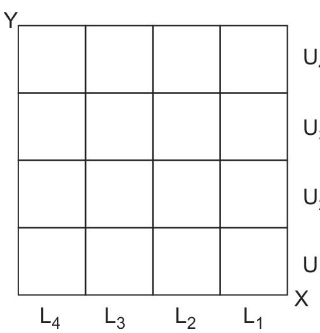
CHAPTER-7 : PERMUTATIONS & COMBINATIONS

UNIT TEST-1

1. The members of a chess club took part in a round robin competition in which each plays every one else once. All members scored the same number of points, except four juniors whose total score were 17.5. How many members were there in the club? (Assume that for each win a player scores 1 point, for draw 1/2 point and zero for losing.)

2. In the given figure you have the road plan of a city. A man standing at X wants to reach the cinema hall at Y by the shortest path. What is the number of different paths that he can take?

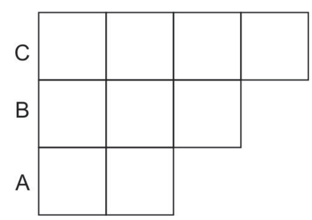


3. There are $2n$ guests at a dinner party. Supposing that the master and mistress of the house have fixed seats opposite one another and that there are two specified guests who must not be placed next to one another find the number of ways in which the company can be placed.

4. The sides AB, BC and CA of triangle ABC have 3, 4 and 5 interior points respectively on them. If the number of triangles that can be constructed using these interior points as vertices is k, then $\left(\frac{k}{5}\right)$ is equal to

5. Seven different coins are to be divided amongst three persons. If no two of the persons receive the same number of coins but each receives atleast one coin and none is left over, then the number of ways in which the division may be made is k, then $\left(\frac{k}{10}\right)$ is equal to.

6. The number of ways in which 5 X's can be placed in the squares of the figure so that no now remains empty is



7. In a shooting competition a man can score 0, 2 or 4 points for each shot. Then the number of different ways in which he can score 14 points in 5 shots is

8. In hockey series between team X and Y, they decide to play till a team wins '10' match. Then the number of ways in which team X wins is $\frac{{}^{20}C_m}{2}$ then m is equal to

Hints and Solutions

1. (a) Set : Form an equation of the total number of points scored using the given information. Solve the equation to find the answer.

Let the number of members be n.

Total number of point $= {}^nC_2$.

$\therefore\ {}^nC_2 - 17.5 = (n-4)\times$ (where x is the number of point scored by each player)

$$n\,(n-1) - 35 = 2\,(n-4)\times$$

$$2x = \frac{n(n-1)-35}{n-4}$$

(where x taken the values 0. 5, 1, 1.5 etc.)

$$= \frac{n^2-n-35}{n-4} \text{ (must be an integer)}$$

$$= \frac{n(n-4)+3(n-4)-23}{n-4} = (n+3) - \frac{23}{n-4}$$

$\Rightarrow \dfrac{23}{n-4}$ must be an integer

$\Rightarrow n - 27$ is the only possibility.

2. (a) If the man moves only in the upward and the leftward direction, then the path will be the shortest.

Use this idea to calculate total number of shortest paths.

As the man wants to travel by one of the many possible shortest paths, he will never turn to the right or turn downward. So a travel by one of the shortest paths is to take 4 horizontal pieces and 4 vertical pieces of roads.

$\therefore$ A shortest path is an arrangement of eight objects $L_1, L_2, L_3, L_4, U_1, U_2, U_3, U_4$ so that the order of L's and U's do not change.

($\therefore$ Clearly L_2 cannot be taken without taking L_1, U_2 cannot be taken without taking U_1, etc.)

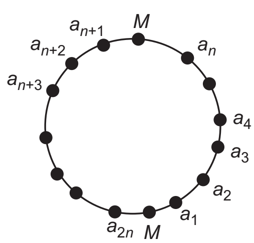
Hence, the number of shortest paths

= The number of arrangements of $L_1, L_2, L_3, L_4, U_1, U_2, U_3, U_4$ where the order of Ls as well as the order of Us do not change

= The number of arrangement treating Ls as identical and Us as identical

$$= \frac{8!}{4!4!} = \frac{8.7.6.5}{24} = 2.7.5 = 70$$

3. (a) Set : Let them and represent seats of the master and misioes respectively and let $a_1, a_2, a_3, \ldots. a_{2n}$ represent the $2n$ seats.



Let the guests who must not be placed next to one another be called P and Q.

Now put P at a_1, and Q at any position, other then a_2, say at a_3; then remaining $2n - 2$ guests can be arranged in the remaining $(2n-2)$positions in $(2n-2)!$ ways. Hence there will be altogether $(2n-2)(2n-2)!$ arrangements of the guests when P is at a_1.

If P is at a_2 there are altogethers $(2n - 3)$ positions for Q hence there will be together $(2n - 3)$ $(2n - 2)!$ arrangements of the guests when P is at a_2.

The same number of arrangement when P is at ^an or $^an + 1$ or ^a2n.

Hence for these positions ($^a1$, ^an, $^an + 1$, ^a2n) of P there are altogether $4\,(2n - 2)(2n - 2)!$ ways.

If P is at a_2 three are altogether $(2n - 3)$ positions for Q hence, there will be together $(2n - 3)$ $(2n - 2)!$ arrangements of the guests when P is at a_2.

The same number of arrangements of the guests when P is at any other position excepting the four positions $^a1$, ^an, $^an + 1$, ^a2n.

Hence for these $(2n - 4)$ positions of P there will be altogether $(2n - 4)(2n - 3)(2n - 2)!$ arrangements of the guests . ...(2)

Hence from (1) and (2), the total no. of ways of arranging the guests

$= 4\,(2n - 2)(2n - 2)! + (2n - 4)\,(2n - 3)(2n - 2)!$

$= (4n^2 - 6n + 4)\,(2n - 2)!$

4. (a) First note in all 3 + 4 + 5, that is, 12 points are used for forming the triangles. Since 3 points are needed to form a triangle, the total number of triangles (including the triangle formed by collinear points and as these collinear points will not form triangles their numbers has to be subtracted from total) is $^{12}C_3 = 220$.

But this includes the number of triangles forned by 3 points on AB = $^3C_3$ = 1, the number of triangles formed by 4 points on BC = $^4C_3$ = 4, and the number of triangles formed by 5 points on CA $^5C_3$ = 10.

Hence the required number of triangles

$$= 220\ (1 + 4 + 10) = 205.$$

$\therefore \quad k = 205$

$\therefore \quad \dfrac{k}{5} = 41$

5. (a) Well with hence people and no. two people getting the same number (and everyone gets at least 1) – the only way is to divide it is 1 – 2 – 4.

Assuming person one gets one coin .... there are 7 possibilities.

Person two gets 2 coins from the remaining 6

$$= \frac{6!}{(6-2)!2!} = 15$$

(because order of the coin does not matter).

Person 3 gets what's left – 4 wins (1 possibility again order does not matter)

But there are also 6 ways to average the people .....

$7 \times 15 \times 6 = 630 \quad \therefore \quad k = 630$

$\therefore \quad \dfrac{k}{10} = 63$

6. (a) Let number of x's in first row be 3, second row be 1, third row be 1.

Number of ways for this is $^4C_3 \times {}^2C_1 \times {}^2C_1 = 16$

Case – (2):

Let number of X's in first row be 2, second row be 1 or 2, third row be 2 or 1 respectively.

Number of ways for this is $2\,({}^4C_2 \times {}^2C_1 \times {}^2C_1) = 24$

Case – (3):

Let number of X's in first row be 1, second row be 2 third row be 2.

Number of ways for this is ${}^4C_1 \times {}^2C_2 \times {}^2C_2 = 4$

Total number of ways is $16 + 24 + 4 = 44$

7. (a) In the shooting competition man can score 0, 2, or 4.

He scored 14 points in 5 shots.

No. of ways in which he can do that,

$\Rightarrow$ (a) $4, 4, 4, 2, 0 = \dfrac{5!}{3!} = 20$

$\Rightarrow$ (b) $2, 2, 2, 4, 4 = \dfrac{5!}{3!2!} = 10$

$\therefore \quad N = 20 + 10$

$\quad = 30$

Hence, the answer is 30.

8. (a) For a team to win the tournament in matches, it has to win 9 matches in $K - 1$ matches and then win the K^{th} match.

Number of ways is

$${}^9C_9 + {}^{10}C_9 + {}^{11}C_9 + {}^{12}C_9 + \ldots\ldots + {}^{18}C_9 = {}^{19}C_{10} = \frac{{}^{20}C_{10}}{2}$$

$\Rightarrow m = 10$